CHAPTERWISE QUESTION

MATHEMATICS

SET A

THREE DIMENSIONAL GEOMETRY

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. The distance of point (2, 5, 7) from the x-axis is

   a) 2 b) $\sqrt{74}$ c) $\sqrt{29}$ d) $\sqrt{53}$

2. *P* is a point on the line segment joining the points (3, 5, –1) and (6, 3, –2). If y-coordinate of point *P* is 2, then its x-coordinate will be

   a) 2 b) $\frac{17}{3}$ c) $\frac{15}{2}$ d) –5

3. Distance of the point (α, β, γ) from Y-axis is

   a) β units b) $|\beta|$ units c) $|\beta| + |\gamma|$ units d) $\sqrt{\alpha^2 + \gamma^2}$ units

4. The direction ratios of 3x + 2y + z + 3 = 0 is

   a) $\left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$ b) $\left(\frac{-3}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$

   c) $\left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$ d) None of these

5. Write the vector equation of the following line : $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

   a) $\vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} - 2\hat{k}\right)$ b) $\vec{r} = \left(6\hat{i} + 7\hat{j} + 7\hat{k}\right) + \lambda\left(4\hat{i} - 4\hat{j} + 2\hat{k}\right)$

   c) $\vec{r} = \left(11\hat{i} + 12\hat{j} + 13\hat{k}\right) + \lambda\left(6\hat{i} + 13\hat{j} - 4\hat{k}\right)$

   d) $\vec{r} = \left(21\hat{i} + 8\hat{j} + 7\hat{k}\right) + \lambda\left(5\hat{i} - 14\hat{j} + 6\hat{k}\right)$

6. If the direction cosines of a line are *k, k* and *k*, then

   a) $k > 0$ b) $0 < k < 1$ c) $k = 1$ d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

7. The vector and cartesian equations of the line through the point (5, 2, –4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$, are

   a) $\vec{r} = \left(3\hat{i} + 2\hat{j} - 8\hat{k}\right) + \lambda\left(5\hat{i} + 2\hat{j} - 4\hat{k}\right); \frac{x-3}{5} = \frac{y-2}{2} = \frac{z+8}{-4}$

   b) $\vec{r} = \left(5\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} - 8\hat{k}\right); \frac{x-3}{5} = \frac{y-2}{2} = \frac{z+8}{-1}$

c) $\vec{r} = \left(5\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} - 8\hat{k}\right); \frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$

d) $\vec{r} = \left(3\hat{i} + 2\hat{j} - 8\hat{k}\right) + \lambda\left(5\hat{i} + 2\hat{j} - 4\hat{k}\right); \frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$

8. What are the direction cosines of a line, which makes equal angles with the coordinate axes?

a) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ b) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$

c) $\pm 2\sqrt{2},\ 3\sqrt{2},\ -3\sqrt{2}$ d) $2\sqrt{3},\ \pm 3\sqrt{5},\ 6\sqrt{5}$

9. A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to

a) 2 b) 1 c) –2 d) –1

For question number 10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

10. Assertion (A) : The points A(2, 9, 12), B(1, 8, 8), C(2, 11, 8), D(1, 12, 12) are the vertices of a rhombus.

Reason (R) : AB = BC = CD + DA and AC = BD.

SECTION - B **2 × 2 = 4**

11. Check whether the lines passing through (1, 1, 2) and (3, 5, 1) is parallel to the line through (4, 2, -1) and (2, -2, 0).

12. Find the vector and Cartesian equation of the line passing through the points (3, -2, -5) and (3, -2, 6).

OR

Find the distance of the point (-2, 4, -5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

SECTION - C **4 × 3 = 12**

13. Find the direction cosines of the sides of the triangle whose vertices are A(3, 5, -4), B(-1, 1, 2) and C(-5, -5, -2).

14. Find the vector and Cartesian equation of a line passing through (-1, 0, 2) and parallel to the vector $2\hat{i} - \hat{j} + 3\hat{k}$.

15. Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

16. Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$

OR

Find the equation of a line passing through the point P(2, –1, 3) and perpendicular to the lines : $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

SECTION - D **2 × 5 = 10**

17. Show that lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection.

18. If a variable line in two adjacent positions has direction cosines *l, m, n* and *l* + δl, *m* + δm, *n* + δn, show that the small angle $\delta\theta$ between two position is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$

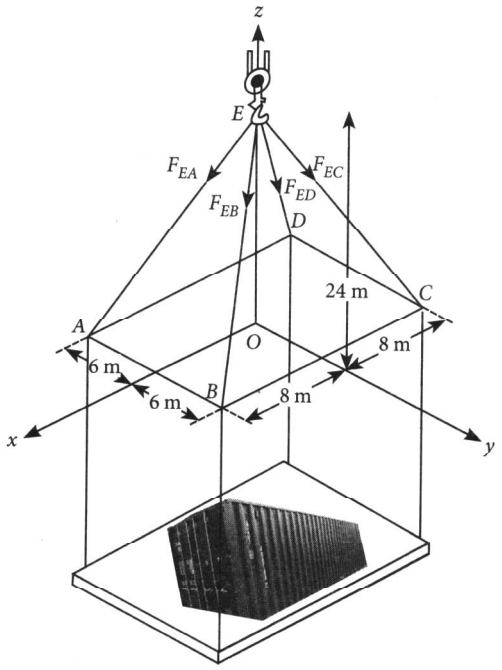
OR

Find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

SECTION - E

Case Study

19. Consider the following diagram, where the forces in the cable are given.



i) Find the Cartesian equation of line along EA. **2**

OR

Find the length of cable EB. Also find which cable the length is equal to EC.

ii) Find the vector $\overline{ED}$ **1**

iii) Find the sum of all vectors along the cable. **1**

CHAPTERWISE QUESTION

MATHEMATICS

SET B

THREE DIMENSIONAL GEOMETRY

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. Write the distance of the point (3, –5, 12) from the x-axis.

   a) 14 units b) 13 units c) 5 units d) 14 units

2. If P (1, 5, 4) and Q(4, 1, –2), find the direction ratios of $\overrightarrow{PQ}$.

   a) 3, –4, –6 b) 6, –5, –2 c) 1, –1, –3 d) 5, 2, 6

3. Find the direction cosines of the line joining the points P(4, 3, –5) and Q(–2, 1, –8).

   a) $\left(\frac{\mp 6}{7}, \frac{\mp 2}{7}, \frac{\mp 3}{7}\right)$ b) $\left(\frac{\mp 2}{7}, \frac{\mp 3}{7}, \frac{\mp 6}{7}\right)$ c) $\left(\frac{\mp 3}{7}, \frac{\mp 2}{7}, \frac{\mp 6}{7}\right)$ d) $\left(\frac{\mp 6}{7}, \frac{\mp 2}{7}, \frac{\mp 4}{7}\right)$

4. The direction ratios of the line passing through two points (2, –4, 5) and (0, 1, –1) is

   a) (–2, 5, –6) b) (–2, 0, –6) c) (–2, 5, 0) d) None of these

5. Write direction ratios of the line x = –3, $\frac{y-1}{3} = \frac{z-2}{-1}$

   a) (0, 4, 2) b) (1, –2, 6) c) 11, 3, –7) d) (0, 3, –1)

6. Find the direction cosines of the line passing through the following points :
   (–2, 4, –5), (1, 2, 3)

   a) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ b) $\frac{3}{\sqrt{44}}, \frac{7}{\sqrt{44}}, \frac{-2}{\sqrt{44}}$

   c) $2\sqrt{2}, -3\sqrt{2}, 8\sqrt{2}$ d) $\frac{3}{\sqrt{27}}, \frac{4}{\sqrt{27}}, -3$

7. Direction ratios of a line are 2, 3, –6. Then direction cosines of a line making obtuse angle with the y-axis are

   a) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ b) $\frac{-2}{7}, \frac{3}{7}, \frac{-6}{7}$ c) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$ d) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$

8. The centroid of a ΔABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, –6), respectively; then the coordinates of the point C is

   a) (1, 1, 3) b) (3, 1, 1) c) (1, 0, 1) d) (1, 1, 2)

9. If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then, find the value of *k*.

   a) $\pm\sqrt{2}$ b) $\pm\sqrt{3}$ c) $\pm 4\sqrt{2}$ d) $\pm 5\sqrt{3}$

For question number 10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

10. Assertion (A) : Cartesian equation of a line AB to $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. The direction cosines $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Reason (R) : If a, b, c are direction ratios of a line, then its direction cosines are $\pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

SECTION - B **2 × 2 = 4**

11. If a line makes angles α, β, γ with the positive directions of the coordinate axes, then calculate the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

12. If A(8, 3, 2), B(7, 1, 4) and C(5, 3, 5) are the vertices of ΔABC. Find $\angle ABC$

SECTION - C **4 × 3 = 12**

13. The equation of a line is 3x + 1 = 6y – 2 = 1 – z. Find the fixed point through which it passes and its direction ratios.

14. Find the shortest distance between the following two lines : $\vec{r} = (1+\lambda)i + (2-\lambda)j + (\lambda+1)\hat{k}$; $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

OR

Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

15. Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) on the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$

16. Find the equation of the line passing through the point P(2, –1, 3) and perpendicular to the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - 3\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

SECTION - D **2 × 5 = 10**

17. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1} \text{ and } z = 2$$

18. Find the distance between the lines L_1 abd L_2 given by $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$

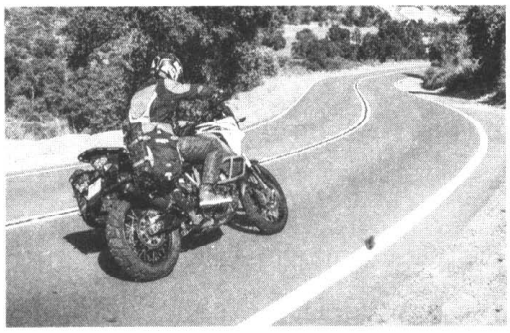
OR

The cartesian equations of a line is 6x – 2 = 3y + 1 = 2z – 2. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through the point (2, –1, –1) which are parallel to the given line.

SECTION - E

Case Study

19. Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.



i) Find the direction cosines of line along motorcycle A is running. **1**

ii) Find the direction ratios of line along which motorcycle B is running. **1**

iii) Find the cartesian equation of the line along which motorcycle A is running. **2**

OR

Find the shortest distance between the given lines.